

# UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Ordinary Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

# 0 4 8 6 5 6 1 6 8 7

#### **ADDITIONAL MATHEMATICS**

4037/11

Paper 1

October/November 2011

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

#### **READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
Total		

This document consists of 14 printed pages and 2 blank pages.



#### Mathematical Formulae

For Examiner's Use

## 1. ALGEBRA

Quadratic Equation

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \cdot$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where *n* is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ .

#### 2. TRIGONOMETRY

*Identities* 

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\csc^2 A = 1 + \cot^2 A$$

Formulae for  $\triangle ABC$ 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^{2} = b^{2} + c^{2} - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 (a) Sets A and B are such that n(A) = 15 and n(B) = 7. Find the greatest and least possible values of

For Examiner's Use

(i)  $n(A \cap B)$ ,

[2]

(ii)  $n(A \cup B)$ .

[2]

**(b)** On a Venn diagram draw 3 sets P, Q and R such that  $P \cap Q = \emptyset$  and  $P \cup R = P$ .

[2]

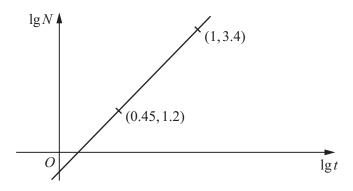
© UCLES 2011 4037/11/O/N/11 **[Turn over** 

The function f is such that  $f(x) = 4x^3 - 8x^2 + ax + b$ , where a and b are constants. It is given that 2x - 1 is a factor of f(x) and that when f(x) is divided by x + 2 the remainder is 20. Find the remainder when f(x) is divided by x - 1.

For Examiner's Use

Variables t and N are such that when  $\lg N$  is plotted against  $\lg t$ , a straight line graph passing through the points (0.45, 1.2) and (1, 3.4) is obtained.





(i) Express the equation of the straight line graph in the form  $\lg N = m \lg t + \lg c$ , where m and c are constants to be found. [4]

(ii) Hence express N in terms of t.

[1]

4	Six-digit numbers are to be formed using the digits 3, 4, 5, 6, 7 and 9. Each digit may only be used once in any number.							
	(i)	Find how many different six-digit numbers can be formed.	[1]	Use				
	Fine	d how many of these six-digit numbers are						
	(ii)		[1]					
	(iii)	greater than 500 000,	[1]					
	(iv)	even and greater than 500 000.	[3]					

A particle moves in a straight line such that its displacement, x m, from a fixed point O at time t s, is given by  $x = 3 + \sin 2t$ , where  $t \ge 0$ .

For Examiner's Use

(i) Find the velocity of the particle when t = 0.

[2]

(ii) Find the value of t when the particle is first at rest.

[2]

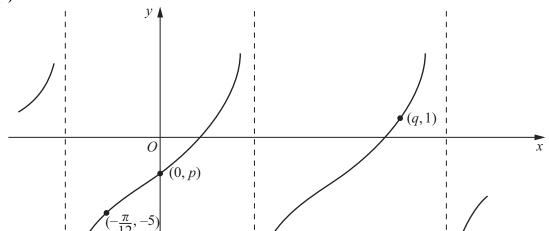
(iii) Find the distance travelled by the particle before it first comes to rest.

[2]

(iv) Find the acceleration of the particle when  $t = \frac{3\pi}{4}$ .

[2]

6 (a)



For Examiner's Use

The diagram shows part of the graph  $y = p + 3\tan 3x$  passing through the points  $(-\frac{\pi}{12}, -5), (0, p)$  and (q, 1). Find the value of p and of q.

(b) It is given that  $f(x) = a \cos(bx) + c$ , where a, b and c are integers. The maximum value of f is 11, the minimum value of f is 3 and the period of f is 72°. Find the value of a, of b and of c.

[4]

- 7 The coefficient of  $x^2$  in the expansion of  $\left(1 + \frac{x}{5}\right)^n$ , where *n* is a positive integer, is  $\frac{3}{5}$ .
- For Examiner's Use

(i) Find the value of n.

[4]

(ii) Using this value of n, find the term independent of x in the expansion of

$$\left(1+\frac{x}{5}\right)^n\left(2-\frac{3}{x}\right)^2.$$
 [4]

8 (a) Find  $\int (e^x + 1)^2 dx$  and hence evaluate  $\int_0^2 (e^x + 1)^2 dx$ . [6]  $\frac{For}{Use}$ 

**(b)** A curve is such that  $\frac{dy}{dx} = (4x+1)^{-\frac{1}{2}}$ . Given that the curve passes through the point with coordinates (2, 4.5), find the equation of the curve. [5]

9 (i) Solve 
$$1 + \cot^2 x = 8 \sin x$$
 for  $0^{\circ} \le x \le 360^{\circ}$ .

For Examiner's Use

[5]

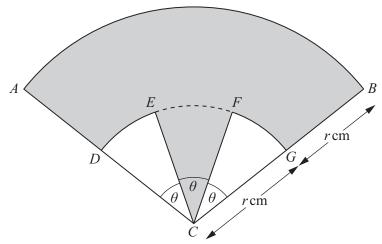
(ii) Solve 
$$4 \sin(2y - 0.3) + 5 \cos(2y - 0.3) = 0$$
 for  $0 \le y \le \pi$  radians.

[5]

Answer only **one** of the following two alternatives.

For Examiner's Use

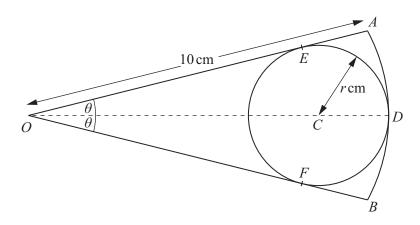
#### **EITHER**



The figure shows a sector ABC of a circle centre C, radius 2r cm, where angle ACB is  $3\theta$  radians. The points D, E, F and G lie on an arc of a circle centre C, radius r cm. The points D and G are the midpoints of CA and CB respectively. Angles DCE and FCG are each  $\theta$  radians. The area of the shaded region is  $5 \,\mathrm{cm}^2$ .

- By first expressing  $\theta$  in terms of r, show that the perimeter, P cm, of the shaded region is given by  $P = 4r + \frac{8}{r}$ . [6]
- Given that r can vary, show that the stationary value of P can be written in the form  $k\sqrt{2}$ , where k is a constant to be found.
- Determine the nature of this stationary value and find the value of  $\theta$  for which it occurs. [2] (iii)

OR



The figure shows a sector OAB of a circle, centre O, radius 10 cm. Angle  $AOB = 2\theta$  radians where  $0 < \theta < \frac{\pi}{2}$ . A circle centre C, radius r cm, touches the arc AB at the point D. The lines OA and OB are tangents to the circle at the points E and F respectively.

(i) Write down, in terms of 
$$r$$
, the length of  $OC$ . [1]

(ii) Hence show that 
$$r = \frac{10 \sin \theta}{1 + \sin \theta}$$
. [2]  
(iii) Given that  $\theta$  can vary, find  $\frac{dr}{d\theta}$  when  $r = \frac{10}{3}$ .

(iii) Given that 
$$\theta$$
 can vary, find  $\frac{dr}{d\theta}$  when  $r = \frac{10}{3}$ . [6]

Given that r is increasing at 2 cms<sup>-1</sup>, find the rate at which  $\theta$  is increasing when  $\theta = \frac{\pi}{6}$ . (iv) [3]

Start your answer to Question 10 here.	Start your answer to Question 10 here.					
Indicate which question you are answering.	EITHER		Examine Use			
	OR					
	•••••					
	•••••					
	•••••					
	•••••					
	•••••					
		•••••				

For Examiner's Use

Continue your answer nere it necessary.

# 15

# **BLANK PAGE**

## **BLANK PAGE**

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.